Load Flow in electrical power systems

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Load Flow analisys

An electric network with *n* buses can be represented by matrix equation:

$$\underline{i} = Y_{bus} \cdot \underline{v}$$

$$\begin{vmatrix} I_1 \\ \vdots \\ I_n \end{vmatrix} = \begin{vmatrix} Y_{11} & \cdots & Y_{1n} \\ \vdots & \ddots & \vdots \\ Y_{n1} & \cdots & Y_{nn} \end{vmatrix} \cdot \begin{vmatrix} V_1 \\ \vdots \\ V_n \end{vmatrix}$$

Where i and v are the current and voltage vectors respectively. Y_{bus} represents the admittance matrix in which Y_{11}-Y_{nn} represents the sum of admittances connected between node and ground, while the other elements represents elements connecting two different nodes

Load Flow analisys

The current in a generic node i can be expressed by:

$$\overline{I_i} = \sum_{k=1}^{n} \overline{V_k} \cdot \overline{Y_{ik}}$$

The complex power at the same node is equal to:

$$\overline{S_i} = \overline{V_i} \cdot \overline{I_i}^* = \overline{V_i} \cdot \sum_{k=1}^{n} \overline{V_i}^* \cdot \overline{Y_{ik}}^* = P_{G_i} - C_i + j\left(Q_{G_i} - D_i\right)$$

Where P_{Gi} and C_i are the produced and adsorbed active power at node i respectively. Q_{Gi} and D_i are the produced and adsorbed reactive power at node i.

Load Flow analisys

The equation of complex power can be also expressed using polar coordinates:

$$P_{G_i} - C_i + j\left(Q_{G_i} - D_i\right) = \sum_{k=1}^{n} V_i \cdot V_k \cdot Y_{ik} \cdot e^{j(\delta_i - \delta_k - \varphi_{ik})}$$

This equation can be dived in real and imaginary part:

$$P_{G_i} - C_i - \sum_{k=1}^{n} V_i \cdot V_k \cdot Y_{ik} \cdot \cos\left(\delta_i - \delta_k - \varphi_{ik}\right) = 0$$

$$Q_{G_i} - D_i - \sum_{k=1}^{n} V_i \cdot V_k \cdot Y_{ik} \cdot \sin\left(\delta_i - \delta_k - \varphi_{ik}\right) = 0$$

This non-linear equation can be solved using Newton-Raphson method to find the module and phase of voltage for each node.

Load Flow analisys

Before to apply the Newton-Raphson method the nodes are divided un three different categories:

- PQ nodes: where are connected pure loads (P_{Gi}=0 , Q_{Gi}=0)
- PV nodes: where are connected generators (the value of P_{Gi} and V_i are known)
- Slack node: balance node of the system (module and phase of the voltage are known)

Load Flow analisys

Starting from these categories of nodes is possible to define the number of equation to calculate module and phase of voltage for each node.

- PQ nodes: introduces 2 equation (the real and imaginary equation of the complex power) because in that nodes module and phase of the voltage are unknown.
- PV nodes: introduces 1 equation (the real equation of the complex power) because in that nodes module is known (imposed by the generator) and phase of the voltage is unknown
- Slack node: doesn't introduce equation

Load Flow analisys

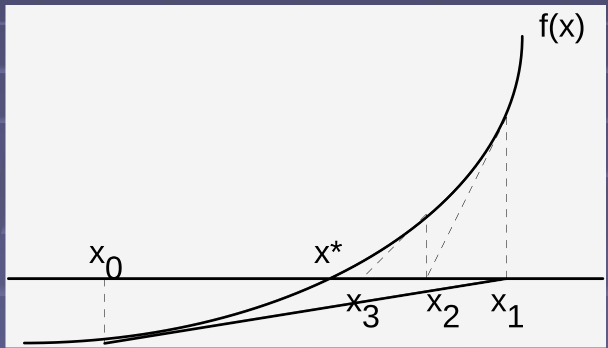
The Newton-Raphson method is a numerical approach that can be used to determine the root of a function f(x) (monodimentional for example).

In particular if is known an approximated value of the solution x^0 is possible to represent the same equation f(x) by its first order Taylor series, around the point x0:

$$f(x) = f\left(x^0\right) + f'\left(x^0\right) \cdot \left(x - x^0\right)$$

Load Flow analisys

This equation represents also the tangent straigth line at the function f(x) in x^0. If we extended the line as far as to intersect the x axes we find a new approximate solution x^1



Starting from this point is possible to realize an iterative procedure to calculate the solution with a stop criteria:

$$x^h = x^{h-1} - \left[f'(x^{h-1})\right]^{-1} f(x^{h-1})$$

$$\left|f\left(x^h\right)\right| < \varepsilon$$

Load Flow analisys

If the function f(x) is multidimensional the first order Taylor series becams equal to:

$$\underline{f}(\underline{x}^{h}) = \underline{f}(\underline{x}^{h-1}) + J_{\underline{x}}^{h-1}(\underline{x}^{h} - \underline{x}^{h-1})$$

Where J is the Jacobian matrix:

$$J_{\underline{x}} = \begin{vmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \cdots & \frac{\partial f_n}{\partial x_n} \end{vmatrix}$$

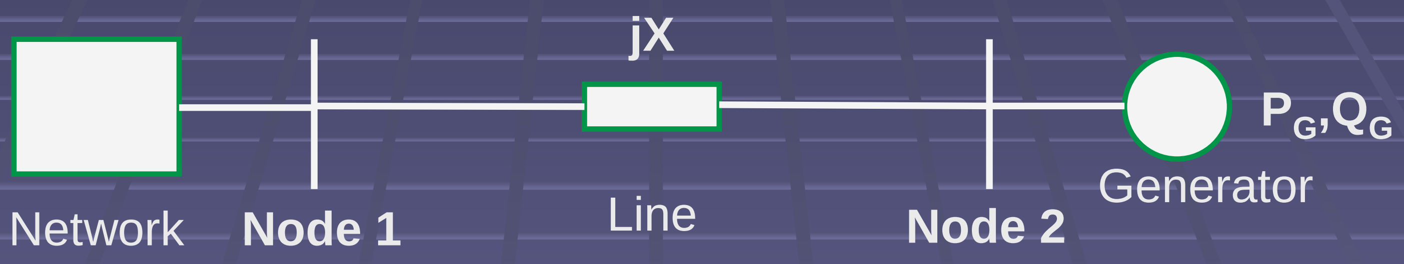
The iterative procedure can be also modified:

$$\underline{x}^{h} = \underline{x}^{h-1} - \left[J_{\underline{x}}^{h-1}\right]^{-1} \underline{f}(\underline{x}^{h-1})$$

$$\max_i \left\{ \left| f_i(\underline{x}^{h}) \right| \right\} < \varepsilon$$

Load Flow analisys

Example: 2 nodes with one generator



$$Y_{bus} = \begin{vmatrix} 0 & -\frac{1}{jX} \\ -\frac{1}{jX} & 0 \end{vmatrix}$$

$$P_{G_i} - C_i - \sum_{k=1}^{n} V_i \cdot V_k \cdot Y_{ik} \cos(\delta_i - \delta_k - \varphi_{ik}) = 0$$

$$P_G - V_1 V_2 \cos\left(\delta_2 + \frac{\pi}{2}\right) = 0$$

DIgSILENT application

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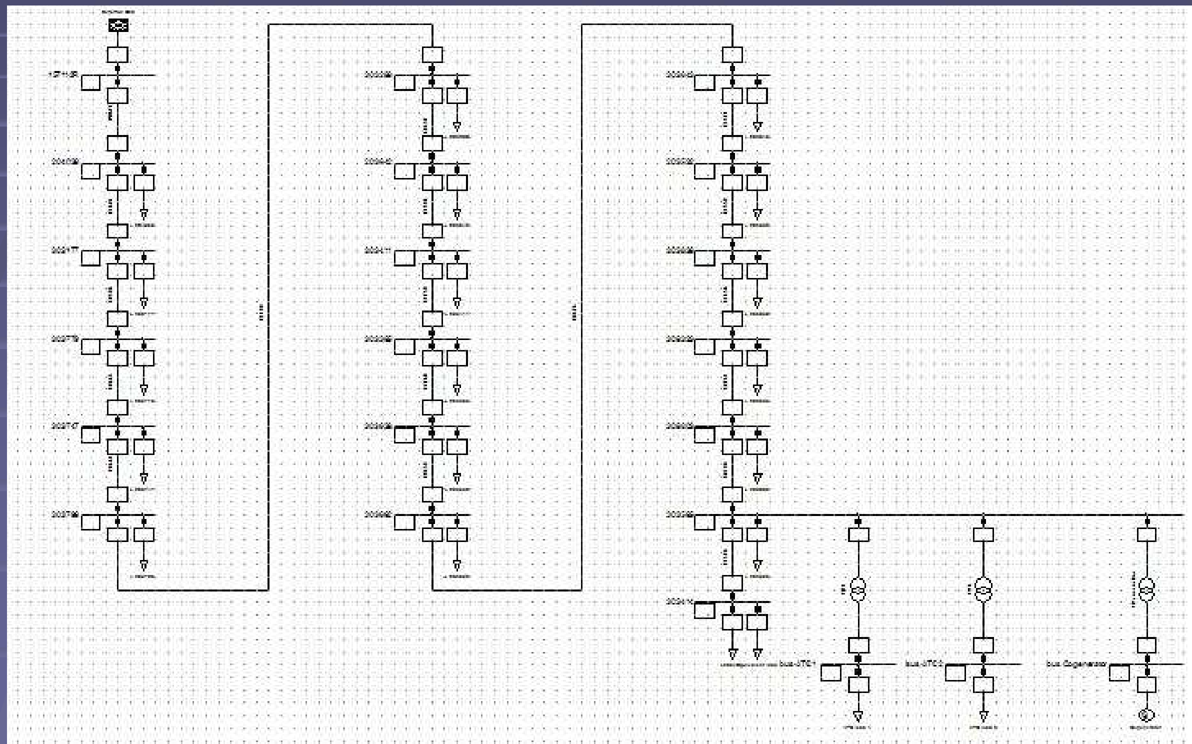
The following exercitation is related to the MV electrical network that supply the Arquata district in Turin.

It will be analyzed the impact of a cogenerator with different electrical power level produced, in the MV network:

- 1 MW (the real situation)
- 10 MW
- 100 MW

DIgSILENT

The DIgSILENT software allows to build all the components which are present in the network



POLYCITY

CONCERTO

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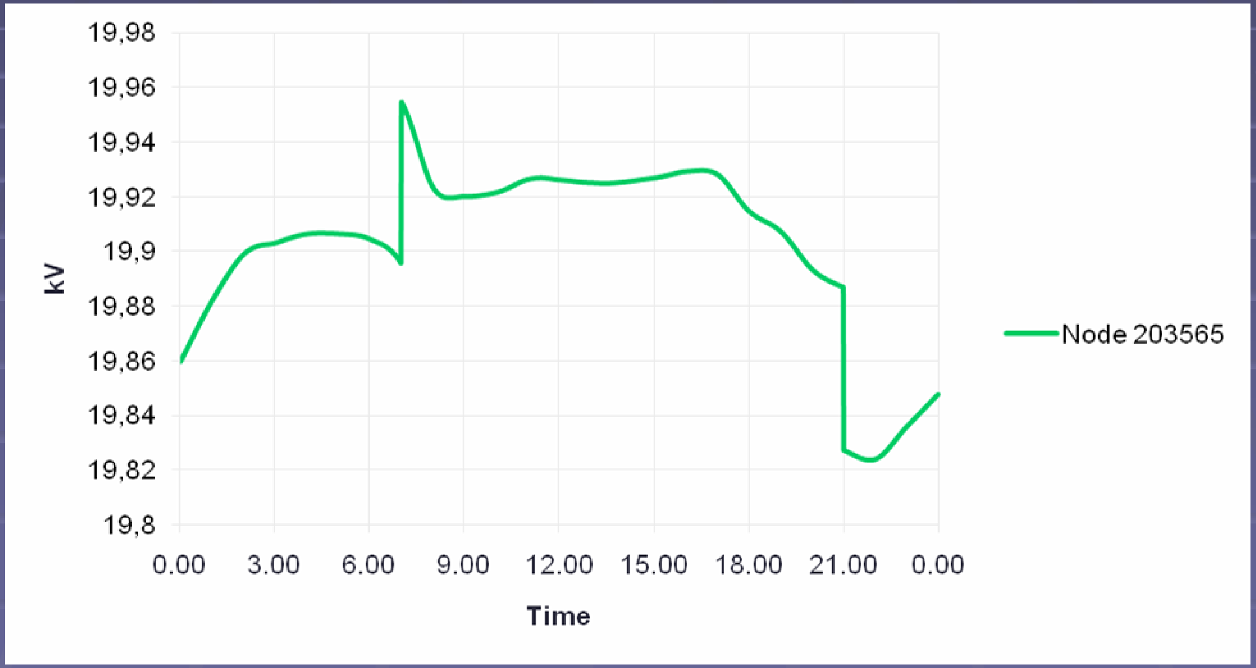
Case 1: 1 MW

Voltage profile node 203565



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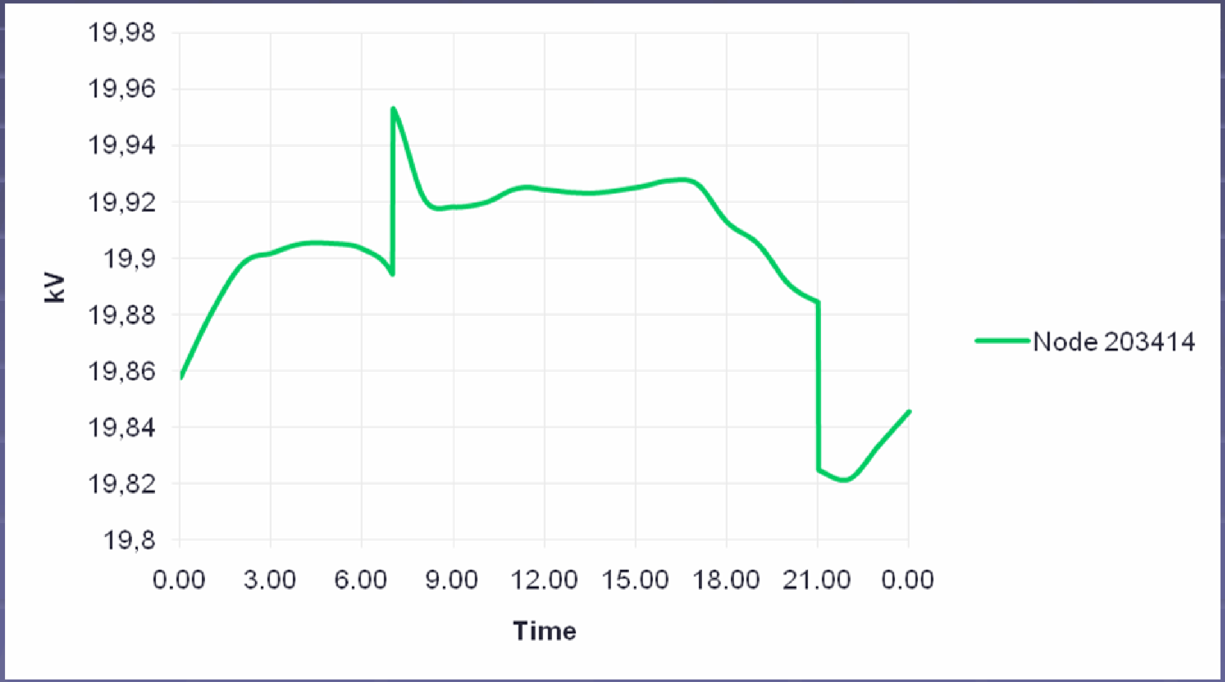
Case 1: 1 MW

Voltage profile node 203414



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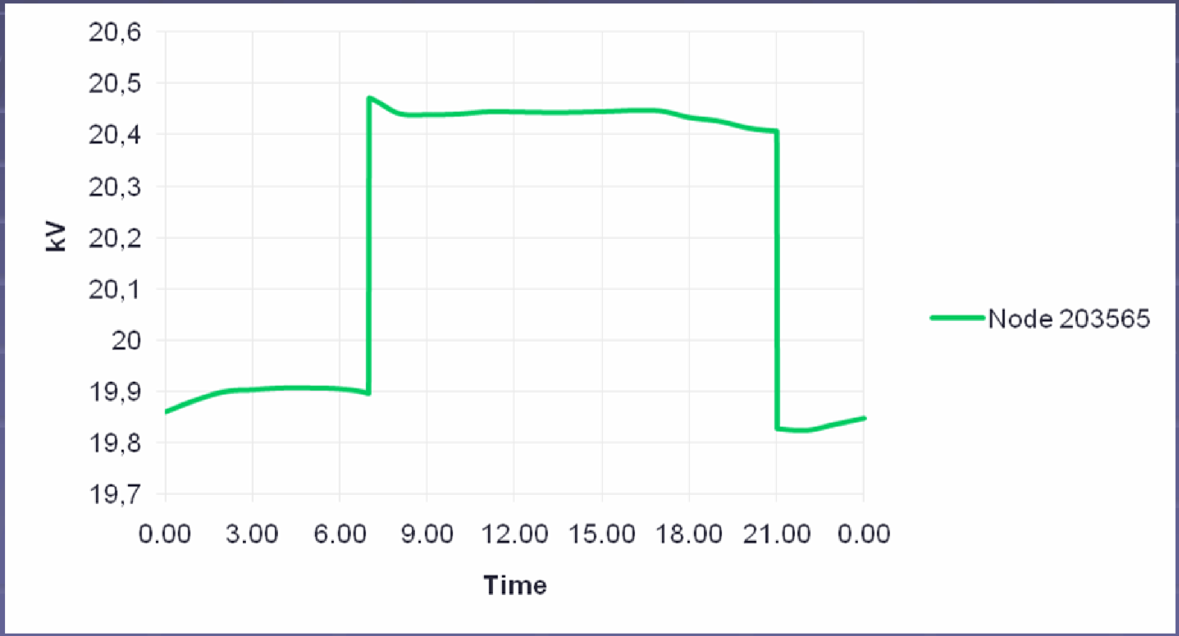
Case 1: 10 MW

Voltage profile node 203565



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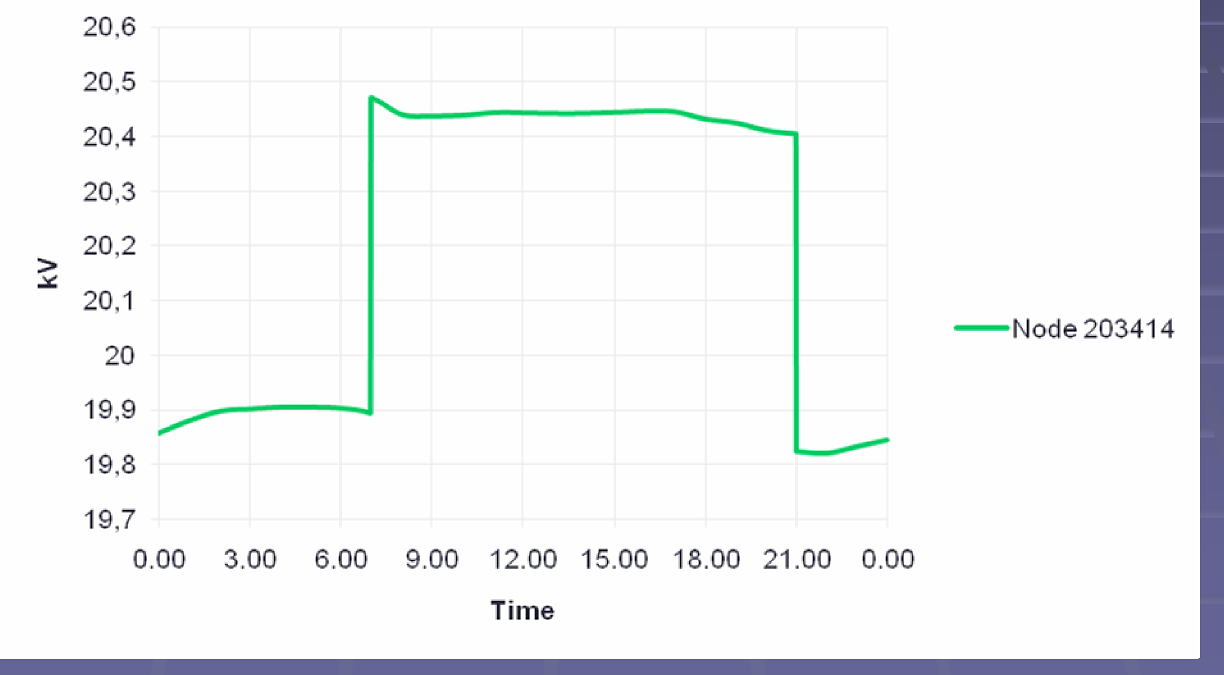
Case 1: 10 MW

Voltage profile node 203414



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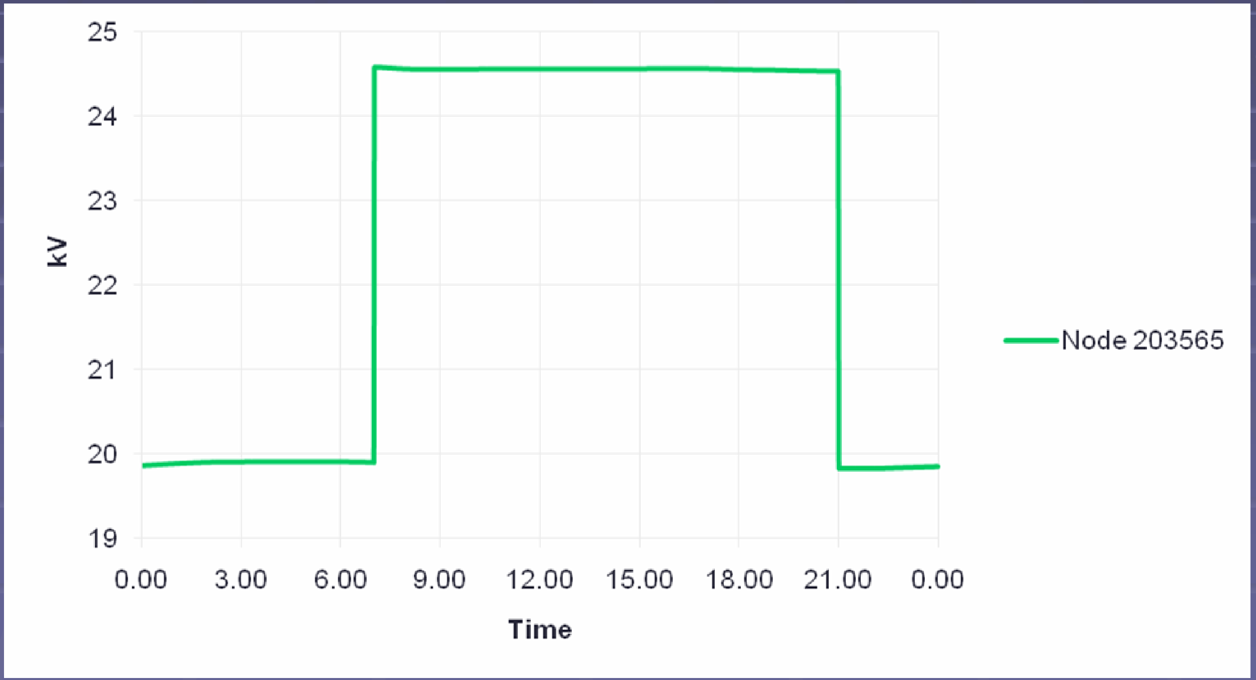
Case 1: 100 MW

Voltage profile node 203565



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Case 1: 100 MW

Voltage profile node 203414